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Charmed scalar mesons and related*

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Charmed scalar mesons are studied. By assigning the newly observed $D_{s0}^+(2.32)$ to the $I_3 = 0$ component, $\hat{F}_I^+$, of iso-triplet four-quark mesons, $\hat{F}_I$'s, which belong to the scalar $[cq][\bar{q}\bar{q}]$, ($q = u, d, s$), multiplet, decay properties of the multiplet members are studied and existence of additional narrow scalar mesons is predicted. Decays of ordinary scalar $\{c\bar{q}\}$ mesons are also studied by comparing with the $K_0^*(1430)$ which has been considered as the scalar $\{n\bar{s}\}$, ($n = u, d$). In addition, it is demonstrated that contributions of four-quark mesons in hadronic weak decays of charm mesons, D and D_s^+, can solve the long standing puzzle in a overall consistent way.

As is well known, the low lying mesons including charm mesons are well described as the quark-antiquark systems. However, the spectrum of the P-wave states was not completed. In particular, in the charm sector, scalar mesons and a part of axial vector mesons were not observed [1]. However, recently, the BABAR collaboration [2] has observed a narrow scalar meson, $D_{s0}^+(2.32)$, with a mass $m_{D_{s0}} \simeq 2.32$ GeV and a width $\Gamma_{D_{s0}} \sim 10$ MeV (a Gaussian fit but its intrinsic width $\lesssim 10$ MeV) in the $D_s^+\pi^0$ channel. The CLEO [3] and the BELLE [4] have confirmed it and observed additionally axial vector $D_{s1}^{*+}(2.46)$ [3, 4] and $D_1^*(2.43)$ [5], and a scalar $D_0^*(2.31)$ [5]. Thus it might be considered that all the possible P-wave $\{c\bar{q}\}$, ($q = u, d, s$) states were completed.

However, since the measured value of $m_{D_{s0}}$ was much lower than the ones predicted by the potential model [6] and the quenched lattice QCD [7], various models to assign it have been proposed: (a) an iso-singlet DK molecule [8], (b) an iso-singlet four-quark $\{cn\bar{s}\bar{n}\}$, ($n = u, d$), meson [9], (c) a mixed state of ordinary $\{c\bar{s}\}$ and iso-singlet $\{cn\bar{s}\bar{n}\}$ [10], (d) an $I_3 = 0$ component $\hat{F}_I^+$ of iso-triplet four-quark $\hat{F}_I \sim [cn][\bar{s}\bar{n}]$, mesons [11, 12], (e) a pole in the unitarized DK amplitude [13], (f) a pole in the DK amplitude based on the chiral Lagrangian [14], etc., in addition to the ordinary scalar $\{c\bar{s}\}$ [15] and the chiral partner of D_s^+ [16] prior to the observation. In the models (a) – (c), the narrow width of the $D_{s0}^+(2.32)$ is automatically satisfied since it has been assigned to iso-singlet states but the other molecules or four-quark states will be broad if they exist and have kinematically allowed strong decay(s). In the model (b), therefore, a narrow peak, i.e., the $D_{s0}^+(2.32)$, might be observed on a broad (about 100 MeV or more) bump arising from an $I_3 = 0$ component of the iso-triplet four-quark mesons in the $D_s^+\pi^0$ mass distribution (if the latter is produced sufficiently). The model (c) seems to be hard to distinguish from the model (a) at the present stage and the mixing between the $\{c\bar{s}\}$ and the $\{cn\bar{s}\bar{n}\}$ seems to be strongly dependent on the hadron dynamics. The model (d) will be studied later in more detail. The model (e) predicts that there exist a scalar DK bound state and a broad resonance dominated by the ordinary scalar $\{c\bar{s}\}$ and that there exist two broad scalar resonances in the $D\pi$ channel. The model (f) which will be studied in the next talk expects existence of a charmed ($C = 1$) scalar meson with an exotic combination of iso-spin and strangeness quantum numbers, $(I, S) = (0, -1)$, in addition to a normal $(I, S) = (0, +1)$ and two $(I, S) = (\frac{1}{2}, 0)$ scalar mesons.

After the observations of the $D_{s0}^+(2.32)$, the mass of the scalar $\{c\bar{s}\}$ system has been calculated from various approaches such as a modified potential model [17], lattice QCD simulations [18, 19], a bag model [20], a quark-meson model [21], a QCD sum rule [22], a light-cone oscillator model [23], a HQET sum rule [24], etc. However, almost all the models still have provided mass values of the scalar $\{c\bar{s}\}$ higher than the measured $m_{D_{s0}} \simeq 2.32$ GeV.

The production rate of $D_{s0}^+(2.32)$ in B decays also has been studied by using the factorization prescription [25] and it has been concluded [26] that it would be nearly equal to that of D_s^+ and larger by about a factor ten than the measured ones if it is the scalar $\{c\bar{s}\}$ state while the rate for a molecule or a four-quark state would be consistent with experiments.

Before studying four-quark mesons, we review very briefly potentials,

$$V_{qq}(\mathbf{r}) = \sum \Lambda_i \Lambda_i v(\mathbf{r}), \quad V_{q\bar{q}}(\mathbf{r}) = -\sum \Lambda_i \Lambda_i v(\mathbf{r}), \qquad (1)$$

between two quarks and between a quark and an antiquark, respectively, mediated by a vector meson with an extra $SU(3)$ degree of freedom [27] corresponding to the color. Although they have been studied much earlier than the discovery of the color [28], the results which are summarized in Table 1 are instructive.

Table 1. Potentials mediated by a vector meson with $SU(3)$ "color".

	qq		$q\bar{q}$	
$SU_c(3)$	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\mathbf{1}$
Potential	$-\frac{8}{3}\langle v\rangle$	$\frac{4}{3}\langle v\rangle$	$\frac{2}{3}\langle v\rangle$	$-\frac{16}{3}\langle v\rangle$

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Table 2. Ideally mixed scalar $[cq][\bar{q}\bar{q}]$ mesons, where S and I denote strangeness and I-spin. The number with (†) is used as the input data.

S	$I=1$	$I=\frac{1}{2}$	$I=0$	Mass(GeV)
1	$\hat{F}_I$		$\hat{F}_0$	2.32(†)
0		$\hat{D}$		2.22
		$\hat{D}^s$		2.42
-1			$\hat{E}^0$	2.32

Now four-quark $\{qq\bar{q}\bar{q}\}$ mesons can be classified into the following four types [29],

$$\{qq\bar{q}\bar{q}\} = \underbrace{[qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q})}_{J^P=0^+,\,1^+,\,2^+} \oplus \underbrace{\{[qq](\bar{q}\bar{q}) \pm (qq)[\bar{q}\bar{q}]\}}_{J^P=1^+}, \quad (2)$$

where the square brackets and the parentheses imply that the wave functions are anti-symmetric and symmetric, respectively, under exchanges of the flavors between them. The first two can have the spin-parity $J^P = 0^+$, 1^+, 2^+ but the last two have only $J^P = 1^+$. However, we are now interested in scalar mesons so that we concentrate on the first two. There are two ways to obtain color singlet four-quark states, i.e., to take the color $SU_c(3)$ $\bar{\mathbf{3}}\times\mathbf{3}$ and $\mathbf{6}\times\bar{\mathbf{6}}$. As seen before, the force between two quarks (or between two antiquarks) is attractive when they are of $\bar{\mathbf{3}}$ (or $\mathbf{3}$) but repulsive when they are of $\mathbf{6}$ (or $\bar{\mathbf{6}}$) while the force between a quark and an antiquark is attractive and much stronger when they are of color singlet. Therefore, it is expected that the scalar $[qq][\bar{q}\bar{q}]$ mesons of $\bar{\mathbf{3}}\times\mathbf{3}$ of $SU_c(3)$ can be the lowest lying four-quark mesons. (However, these two, i.e., $\bar{\mathbf{3}}\times\mathbf{3}$ and $\mathbf{6}\times\bar{\mathbf{6}}$ of the color $SU_c(3)$, can mix with each other.) In fact, the MIT bag model with the bag potential and a spin-spin force shows that the $[qq][\bar{q}\bar{q}]$ mesons which make a $\mathbf{9}$-plet of the flavor $SU_f(3)$ are the lowest lying states [29]. Such a multiplet may be realized by the observed $a_0(980)$, $f_0(980)$, $\sigma(600)$ [1] and $\kappa(800)$ [30] as suggested in Ref. [29]. To distinguish four-quark mesons with the same flavor and spin but different color configurations (dominantly $\bar{\mathbf{3}}\times\mathbf{3}$ of lower mass and dominantly $\mathbf{6}\times\bar{\mathbf{6}}$ of higher mass), we put $*$ on the symbols of the heavier class of four-quark mesons in accordance with Ref. [29].

Replacing one of q's in the $[qq][\bar{q}\bar{q}]$ by the c quark, we can obtain scalar $[cq][\bar{q}\bar{q}]$ mesons. We list the lighter class [dominantly $\bar{\mathbf{3}}\times\mathbf{3}$ of $SU_c(3)$] of ideally mixed $[cq][\bar{q}\bar{q}]$ mesons in Table 2, where the mass values have been estimated by assigning the newly observed $D_{s0}^+(2.32)$ to the $\hat{F}_I^+$, using the quark counting with $\Delta_s = m_s - m_n \simeq 0.1$ GeV and $m_{\hat{F}_I} = m_{D_{s0}} \simeq 2.32$ GeV as the input data. The $\hat{F}_I$ and $\hat{F}_0$ are the $I=1$ and $I=0$ components, respectively. The $\hat{D}$ and $\hat{D}^s$ are two different iso-doublets, where the latter contains an $(s\bar{s})$ pair. The $\hat{E}^0$ is the exotic scalar meson with $C=1$ and $S=-1$, i.e., $\hat{E}^0 \sim [cs][\bar{u}\bar{d}]$.

We now study decay rates of the above $[cq][\bar{q}\bar{q}]$ mesons. The rate for the decay, $A(\mathbf{p}) \to B(\mathbf{p}') + \pi(\mathbf{q})$, is given by

$$\Gamma(A \to B+\pi) = \left(\frac{1}{2J_A+1}\right)\left(\frac{q_c}{8\pi m_A^2}\right) \times \sum_{spins} |M(A \to B+\pi)|^2, \quad (3)$$

where J_A, q_c and $M(A \to B+\pi)$ denote the spin of the parent A, the center-of-mass momentum of the final B and π mesons and the decay amplitude, respectively. To calculate the amplitude, we use the PCAC (partially conserved axial-vector current) hypothesis and a hard pion approximation in the infinite momentum frame (IMF), i.e., $\mathbf{p} \to \infty$ [31]. In this approximation, the amplitude is evaluated at a little unphysical point, i.e., $m_\pi^2 \to 0$. By assuming that the q^2 dependence of the amplitude is mild as was in the old current algebra [32], it is given by

$$M(A \to B+\pi) \simeq \left(\frac{m_A^2 - m_B^2}{f_\pi}\right)\langle B|A_{\bar{\pi}}|A\rangle, \quad (4)$$

where A_π is the axial counterpart of the isospin, $I(=V_\pi)$. The *asymptotic matrix element* of A_π (matrix elements of A_π taken between single hadron states with infinite momentum), $\langle B|A_\pi|A\rangle$, gives the dimensionless $AB\pi$ coupling strength.

We parameterize later the asymptotic matrix elements of A_π and A_K using the asymptotic flavor symmetry, which is, roughly speaking, flavor symmetry of asymptotic matrix elements. (Asymptotic flavor symmetry and its fruitful results were reviewed in Ref. [31].) However, the asymptotic flavor symmetry may be broken. The measure of the (asymptotic) flavor symmetry breaking is given by the form factor, $f_+(0)$'s, of related vector currents at the zero momentum transfer squared ($q^2=0$). The estimated values of $f_+(0)$'s are

$$f_+^{(\pi K)}(0) = 0.961 \pm 0.008, \quad (5)$$

$$f_+^{(\bar{K}D)}(0) = 0.74 \pm 0.03, \quad (6)$$

$$\frac{f_+^{(\pi D)}(0)}{f_+^{(\bar{K}D)}(0)} = 1.00 \pm 0.11 \pm 0.02, \quad (7)$$

$$= 0.99 \pm 0.08, \quad (8)$$

where the above values of the form factors, Eqs.(5) – (8), have been given in Refs. [33] – [36], respectively. They suggest that the asymptotic flavor $SU_f(3)$ symmetry works well while the asymptotic $SU_f(4)$ is broken to the extent of 20 – 30 %. In fact, the asymptotic $SU_f(4)$ symmetry has predicted the rates [31, 37], $\Gamma(D^{*+} \to D^0\pi^+) \simeq 96$ keV and $\Gamma(D^{*+} \to D^+\pi^0) \simeq 42$ keV, which are larger by about 40 % than $\Gamma(D^{*+} \to D^0\pi^+) = 65\pm18$ keV and $\Gamma(D^{*+} \to D^+\pi^0) = 30 \pm 8$ keV from the measured decay width [38], $\Gamma_{D^{*\pm}} = 96 \pm 4 \pm 22$ keV, and the branching fractions compiled in Ref. [1]. The above suggests that the size of the asymptotic matrix elements

Table 3. Dominant decays of scalar $[cq][\bar{q}\bar{q}]$ mesons and their estimated widths. The measured width, $\Gamma(\hat{F}_I^+ \to D_s^+\pi^0) \sim 10$ MeV, is used as the input data. The decays into the final states between angular brackets are not allowed kinematically as long as the parent mass values in the parentheses are taken.

Parent (Mass in GeV)	Final State	Width (MeV)
$\hat{F}_I^{++}(2.32)$	$D_s^+\pi^+$	
$\hat{F}_I^{+}(2.32)$	$D_s^+\pi^0$	10
$\hat{F}_I^{0}(2.32)$	$D_s^+\pi^-$	
$\hat{D}^+(2.22)$	$D^0\pi^+$	10
	$D^+\pi^0$	5
$\hat{D}^0(2.22)$	$D^+\pi^-$	10
	$D^0\pi^0$	5
$\hat{D}^{s+}(2.42)$	$D^+\eta$	–
$\hat{D}^{s0}(2.42)$	$D^0\eta$	–
$\hat{F}_0^+(2.32)$	$< D_s^+\eta >$	–
	$D_s^+\pi^0$	(I-spin viol.)
$\hat{E}^0(2.32)$	$< D\bar{K} >$	–

of axial charge A_π between charmed meson states will be smaller by about 20 % than the ones in the asymptotic symmetry limit.

Asymptotic matrix elements including four-quark meson states have been parameterized previously [39, 40, 41] by using asymptotic flavor $SU_f(3)$ symmetry. We here list the related ones,

$$\begin{aligned}&\langle D_s^+|A_{\pi^-}|\hat{F}_I^{++}\rangle = \sqrt{2}\langle D_s^+|A_{\pi^0}|\hat{F}_I^{+}\rangle = \langle D_s^+|A_{\pi^+}|\hat{F}_I^{0}\rangle\\ &= -\langle D^0|A_{\pi^-}|\hat{D}^+\rangle = 2\langle D^+|A_{\pi^0}|\hat{D}^+\rangle\\ &= -2\langle D^0|A_{\pi^0}|\hat{D}^0\rangle = -\langle D^+|A_{\pi^+}|\hat{D}^0\rangle. \qquad (9)\end{aligned}$$

Inserting Eq.(4) with Eq.(9) into Eq.(3), we can calculate approximate rates for the allowed two-body decays. Here we equate the calculated rate for the $\hat{F}_I^+ \to D_s^+\pi^0$ decay to the measured width of the $D_{s0}^+(2.32)$, i.e., $\Gamma(\hat{F}_I^+ \to D_s^+\pi^0) \sim 10$ MeV, since we do not find any other decays which can have sizeable rates, and then use it as the input data when we estimate the rates for the other decays. The results are listed in Table 3. However, the numerical values should not be taken too literally since the intrinsic width of the $D_{s0}^+(2.32)$ as the input data is still not definite. (We expect that it will be in the region between a few and ~ 10 MeV. Such a narrow width is understood by the small overlapping of wavefunctions between the initial and final states [12].) The calculated widths of $\hat{F}_I$ and $\hat{D}$ are in the region, $\sim (10 - 15)$ MeV, so that they will be observed as narrow resonances in the $D_s^+\pi$ and $D\pi$ channels, respectively. The mass of $\hat{D}^s$ is approximately on the threshold of the iso-spin conserving $\hat{D}^s \to D\eta$, so that it is not clear if they are kinematically allowed. Besides, the decays are sensitive to the η-η' mixing scheme which is still model dependent [42]. Therefore, we need more precise and reliable values of $m_{\hat{D}^s}$, η-η' mixing parameters and decay constants of η to obtain a definite result. The $[cq][\bar{q}\bar{q}]$ multiplet contains the exotic state $\hat{E}^0$ with $C = -S = 1$ whose mass is expected to satisfy approximately $m_{\hat{F}_I} \simeq m_{\hat{F}_0} \simeq m_{\hat{E}^0}$ from the simple quark counting as in Table 2. If it is the case, the $\hat{E}^0$ cannot decay through strong interactions or through electromagnetic interactions but only through weak interactions [43].

We now study decay widths of the ordinary scalar $\{c\bar{s}\}$ and $\{c\bar{n}\}$ mesons comparing with the $K_0^*(1.43)$ which has been considered as the $^3P_0$ $\{n\bar{s}\}$ state [44]. Substituting the measured values [1], $\Gamma(K_0^* \to all) = 294 \pm 23$ MeV and $\mathrm{Br}(K_0^* \to K\pi) = 93 \pm 10$ %, into Eq.(3) and using Eq.(4), we obtain $|\langle K^+|A_{\pi^+}|K_0^{*0}\rangle| \simeq 0.29$, where we have used the iso-spin $SU_I(2)$ symmetry which is always assumed in this talk. In the asymptotic $SU_f(4)$ symmetry limit [31, 45], we obtain

$$\begin{aligned}&\langle D^+|A_{\pi^+}|D_0^{*0}\rangle = 2\langle D^+|A_{\pi^0}|D_0^{*+}\rangle\\ &= -2\langle D^0|A_{\pi^0}|D_0^{*0}\rangle = \langle D^0|A_{K^-}|D_{s0}^{*+}\rangle\\ &= \langle D^+|A_{\bar{K}^0}|D_{s0}^{*+}\rangle = \langle K^+|A_{\pi^+}|K_0^{*0}\rangle, \qquad (10)\end{aligned}$$

where $D_0^* \sim \{c\bar{n}\}$ and $D_{s0}^{*+} \sim \{c\bar{s}\}$. (When we take account of the about 20 % breaking of the asymptotic $SU_f(4)$ symmetry,) the sizes of the above asymptotic matrix elements are estimated as

$$\begin{aligned}&|\langle D^+|A_{\pi^+}|D_0^{*0}\rangle| = |2\langle D^+|A_{\pi^0}|D_0^{*+}\rangle|\\ &= |-2\langle D^0|A_{\pi^0}|D_0^{*0}\rangle| = |\langle D^0|A_{K^-}|D_{s0}^{*0}\rangle|\\ &= |\langle D^+|A_{\bar{K}^0}|D_{s0}^{*+}\rangle|\\ &= |\langle K^+|A_{\pi^+}|K_0^{*0}\rangle|(\times 0.8) \simeq 0.23. \qquad (11)\end{aligned}$$

It is expected that a sum of the rates for the $D_0^{*0} \to D^+\pi^-$ and $D^0\pi^0$ decays saturates approximately the total decay rate of D_0^{*0}. The iso-spin symmetry leads to $\Gamma(D_0^{*0} \to (D\pi)^0) = \Gamma(D_0^{*+} \to (D\pi)^+)$. The decays, $D_{s0}^{*+}(2.45) \to (DK)^+$'s, also saturate approximately the total decay rate of D_{s0}^{*+}. If we take tentatively $m_{D_0^*} \simeq 2.35$ GeV and $m_{D_{s0}^*} \simeq m_{D_0^*} + \Delta_s \simeq 2.45$ GeV which are around the average values predicted by the potential model [6] and the quenched lattice QCD [7], we can obtain

$$\begin{aligned}&\Gamma_{D_0^*(2.35)} \simeq 90\ [\times(0.8)^2]\ \mathrm{MeV},\\ &\Gamma_{D_{s0}^{*+}(2.45)} \simeq 70\ [\times(0.8)^2]\ \mathrm{MeV}, \qquad (12)\end{aligned}$$

where we have replaced π by K in Eqs.(3) and (4) when we obtain the second equation.

Although the BELLE collaboration [5] has recently reported that a charmed scalar resonance $D_0^0(2.31)$ with a mass 2308 ± 60 MeV and a width 279 ± 99 MeV has been observed in the $D^+\pi^-$ channel and claimed that the result is consistent with the conventional $D_0^{*0} \sim \{c\bar{u}\}$ state,

some comments on the above result and claim are now in order [46]. In Ref. [5], it has been tried to fit four different model amplitudes to the measured $D^+\pi^-$ mass distribution and to search for the most likely solution. In all the amplitudes, however, only one scalar meson pole has been taken into account so that the χ^2 value has been not sufficiently small even in the most likely solution which provided the above mass and width. In particular, significant deviations between the most likely solution and the measured mass distribution are seen in the broad scalar meson region. Therefore, it is expected that a much better fit to the measured $D\pi$ mass distribution will be obtained if an extra scalar meson pole is additionally taken into account in the model amplitude.

The fitted mass value $m_{D_0} \simeq 2.31$ GeV of the scalar meson $D_0^0(2.31)$ is a little lower than the one of D_0^* from a quenched relativistic lattice QCD [7] and an unquenched but static one [18], $m_{D_0^*}(\text{lattice}) \sim 2.33$ GeV, while it is much lower than the one from the potential models [6, 17], $m_{D_0^*}(\text{potential}) \sim 2.4$ GeV. On the other hand, it is is too high when it is compared with the previously observed $D_{s0}^+(2.32)$. Namely, if it is assumed that these two are the ${}^3P_0$ $\{c\bar{s}\}$ and $\{c\bar{n}\}$, it is not natural that they are approximately degenerate but the mass difference between them should be nearly equal to $\Delta_s \simeq 100$ MeV. (It is hard to expect the above degeneracy, $m_{D_{s0}} - m_{D_0} \ll \Delta_s$, unless they have partners, i.e., extra scalar meson(s), to mix with and unless the dynamics of the mixings are very much different from each other. However, in the case of Ref. [5], the model amplitudes include only one scalar meson pole but no extra scalar meson as the partner to mix with.) So, it is natural to consider that these two have different structure, for example, one is the ordinary $\{c\bar{q}\}$ and the other is a four-quark or a molecule.

The width of $D_0(2.31)$ given by the BELLE collaboration was anomalously broad since, as seen before, the width of D_0^* has been expected to be $\Gamma_{D_0^*} \sim (60-90)$ MeV by comparing with the $K_0^*(1.43)$ which has been considered as the ${}^3P_0$ $\{n\bar{s}\}$ state. Therefore our scenario [46] is that there coexist two scalar states in the region of the broad bump around 2.31 GeV observed by the BELLE [5] and that the heavier one is the ordinary ${}^3P_0$ $\{c\bar{n}\}$ with a mass ~ 2.35 GeV and a width $\sim (60-90)$ MeV and the other is the four-quark $\hat{D} \sim [cn][\bar{n}\bar{n}]$ with $m_{\hat{D}} \simeq 2.22$ GeV and $\Gamma_{\hat{D}} \sim 15$ MeV [11]. It is desired that the BELLE collaboration will reanalyze the measured $D\pi$ mass distribution by using a model amplitude with two scalar meson poles. The strange counterpart D_{s0}^{*+} of the D_0^* will be around 2.45 GeV and its dominant decays are $D_{s0}^{*+} \to D^0K^+$ and D^+K^0 so that its width is $\sim (40-70)$ MeV as seen before. It will be observed as an ordinary resonance around 2.45 GeV in $(DK)^+$ mass distributions with high statistics.

So far we have studied classification of the new resonances and strong decays of the members of the multiplet including them. For the production rate of the new resonance, $D_{s0}^+(2.32)$, in B-decays, we have referred to a review paper [26] which has concluded that the factorization provides too big rates if the $D_{s0}^+(2.32)$ is the scalar $\{c\bar{s}\}$ while, if it is a four-quark or a molecule, its production rate is consistent with experiments, i.e., the production rate for the four-quark mesons in B-decays will be much smaller than that of the $\{c\bar{s}\}$ mesons. Therefore, we expect that the existence of four-quark mesons will be confirmed in experiments with higher statistics in future, although we have no evidence for a peak in the $D_s^+\pi^\mp$ [47, 48], $D^0\pi^\mp$ and $D^+\pi^\mp$ mass distributions [48] at the present stage.

Now we study a possible role of scalar four-quark $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$ mesons in hadronic weak decays of charm mesons. For our basic idea on hadronic weak interactions, see Ref. [49], although hadronic weak decays of B mesons have been studied in the paper. For our technical details to study hadronic weak decays of charm mesons, see Ref. [41] and references quoted therein. We start with the assumption that the decay amplitude can be given by a sum of factorizable and non-factorizable ones. The factorizable amplitude is calculated in the so-called BSW scheme [50] in which, to apply the factorization, the BSW Hamiltonian, H_w^{BSW}, has to be prepared by applying the Fierz reshuffling to the conventional effective weak Hamiltonian, H_w,

$$H_w \to H_w^{\text{BSW}} + \tilde{H}_w, \tag{13}$$

where

$$\begin{gathered}
H_w \simeq \frac{G_F}{\sqrt{2}}\Big\{c_1 Q_1^{(s'c)} + c_2 Q_2^{(s'c)} + \cdots\Big\} + h.c., \\
H_w^{\text{BSW}} \simeq \frac{G_F}{\sqrt{2}}\Big\{a_1 Q_1^{(s'c)} + a_2 Q_2^{(s'c)} + \cdots\Big\} + h.c., \\
a_1 = c_1 + \frac{c_2}{N_c} \gg a_2 = c_2 + \frac{c_1}{N_c}, \\
Q_1^{(s'c)} =: (\bar{u}d')_L(\bar{s}'c)_L :, \quad Q_2^{(s'c)} =: (\bar{s}'d')_L(\bar{u}c)_L :
\end{gathered}$$

with $(\bar{q}q)_L = \bar{q}\gamma_\mu(1-\gamma_5)q$. Here c_1 and c_2 are the Wilson coefficients with hard gluon corrections, and N_c the color degree of freedom. As seen in Eq. (13), we inevitably have an extra term, $\tilde{H}_w$, which is given by a color singlet sum of colored current products, i.e.,

$$\begin{gathered}
\tilde{H}_w \simeq \frac{G_F}{\sqrt{2}}\Big\{c_2 \tilde{Q}_1^{(s'c)} + c_1 \tilde{Q}_2^{(s'c)} + \cdots\Big\} + h.c., \\
\tilde{Q}_1^{(s'c)} = 2\sum_a :(\bar{u}t^a d')_L(\bar{s}'t^a c)_L :, \\
\tilde{Q}_2^{(s'c)} = 2\sum_a :(\bar{s}'t^a d')_L(\bar{u}t^a c)_L :,
\end{gathered}$$

when we obtain the H_w^{BSW}, where t^a's are the generators of the color $SU_c(3)$. Although it has been taken away in the BSW scheme, we consider that it provides the non-factorizable amplitude which will be controlled by dynamics of hadrons and that the non-factorizable amplitude can play an important role in hadronic weak interactions of K, charm mesons and some of B decays in which some selection rules such as the color (and/or

Table 4. Branching ratios (%) for the $D \to PP$, ($P = \pi$, K) decays. (1) is given by the factorized amplitudes only, (2) by a sum of factorized and non-factorizable amplitudes, where the latter contains the continuum contribution and the poles of the glue-rich scalar and the scalar hybrid meson, and (3) by the four-quark meson pole amplitudes in addition to the ones in (2).

Decays	(1)	(2)	(3)	$\mathcal{B}_{\rm exp}$
$D^+ \to \bar{K}^0\pi^+$	3.27	1.06	2.72	2.71 ± 0.20
$D^0 \to K^-\pi^+$	2.41	9.19	3.83	3.83 ± 0.09
$D^0 \to \bar{K}^0\pi^0$	0.00	3.71	2.31	2.30 ± 0.22
$D_s^+ \to \bar{K}^0K^+$	0.20	7.28	3.50	3.6 ± 1.1
$D^0 \to \pi^-\pi^+$	0.15	0.34	0.14	0.143 ± 0.007
$D^0 \to \pi^0\pi^0$	0.00	0.11	0.09	0.084 ± 0.022
$D^+ \to \pi^0\pi^+$	0.12	0.12	0.23	0.25 ± 0.07
$D^0 \to K^-K^+$	0.19	0.68	0.42	0.412 ± 0.014
$D^0 \to \bar{K}^0K^0$	0.00	0.04	0.04	0.071 ± 0.019
$D^+ \to \bar{K}^0K^+$	0.47	1.00	0.52	0.57 ± 0.06
$D_s^+ \to \pi^+K^0$	0.17	0.17	0.05	< 0.8
$D_s^+ \to \pi^0K^+$	0.00	0.07	0.06	—

helicity) suppressions work. We will estimate contributions of the non-factorizable amplitudes using a hard pion technique [31, 51] in the infinite momentum frame which is an innovation of the old current algebra [32]. In this approximation, the non-factorizable amplitude is given by a sum of all possible pole amplitudes and the so-called equal-time commutator (ETC) term which arises from the continuum contribution mediated by multi-hadron intermediate states [32]. Among the possible pole amplitudes, contributions of the ordinary excited meson states are neglected since wave function overlappings between the excited states and the external states in two body decays of charm mesons under consideration will be small and, in particular, the value of wavefunction of orbitally excited state at the origin, $\Psi(0)_{L\neq 0}$, is expected to be small. In the u-channel, all contributions of the excited states are neglected since their contributions are expected to be small, while, in the s-channel, a part of (the heavier class of) scalar four-quark mesons can contribute to the s-channel pole amplitudes of the spectator decays (if they exist) since the related four-quark mesons are expected to have their masses close to the parent charm masses, m_D and m_{D_s}. For example, the mass of $\hat{\sigma}^{s*}$ which can contribute to the $D^0 \to K^+K^-$ is expected to be very close to m_D and therefore it can play an important role in the $D^0 \to K^+K^-$ decay. However, it cannot contribute to the $D^0 \to \pi^+\pi^-$ in which the corresponding pole is given by $\hat{\sigma}^*$ but $m_{\hat{\sigma}^*} \ll m_D$, so that, in the latter, the $\hat{\sigma}^*$ contribution would be less important. In this way, we may be able to find a solution to the long standing puzzle [1],

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 2.88 \pm 0.15, \qquad (14)$$

in a overall consistent way [41]. In the annihilation decays (in the weak boson mass $m_W \to \infty$ limit), scalar hybrid $\{q\bar{q}g\}$ mesons can contribute to their s-channel pole amplitudes. The penguin term can induce scalar glue-ball (or glue-rich scalar meson) contributions. The factorized and non-factorizable amplitudes, in which contributions of hybrid mesons were not taken into account, have been given explicitly in Ref. [41].

Although the amplitudes obtained in this way includes many unknown parameters, we evaluated numerically and compared with experiments assuming that the form factors, $f_+(0)$'s, of charm changing vector currents satisfy the $SU_f(3)$ symmetry as seen before and their q^2 dependence is given by a monopole form as usual, taking the measured value of $f_+^{(\bar{K}D)}(0)$, using (and changing a little) the mass values of four-quark mesons predicted in Ref. [29] and treating other parameters as adjustable ones, and then reproduced fairly well the measured branching ratios for two body decays of charm mesons [41]. We now improve the above result taking additionally account of contributions of scalar hybrid mesons which have not been considered in our previous studies. Before doing this, we list the parameters involved: the form factor, $f_+^{(\bar{K}D)}(0)$; the coefficients, a_1 and a_2, in the $H_w^{\rm BSW}$ at the scale $\mu \sim m_c$ which might be some what different from the ones given by the perturbative QCD because of so-called final state interactions controlled by dynamics of hadrons [49]; the asymptotic matrix element, $\langle\pi^+|\tilde{H}_w|D_s^+\rangle$; the parameters, k_a^*, k_s^*, k_H and f_g, describing pole contributions (given by products of asymptotic matrix elements of A_π and $\tilde{H}_w$, $\langle\pi^+|A_\pi|n\rangle\langle n|\tilde{H}_w|D_s^+\rangle$, in the unit of $\langle\pi^+|\tilde{H}_w|D_s^+\rangle$) of $n = [qq][\bar{q}\bar{q}]$, $(qq)(\bar{q}\bar{q})$, $\{q\bar{q}g\}$ (scalar hybrid mesons) and S^* (the glue-rich scalar meson), respectively; the relative phase δ between the factorized and the non-factorizable amplitudes; the phases arising from non-resonant meson-meson interactions, $\delta(\pi\pi)_0$, $\delta(K\bar{K})_0$, $\delta(K\bar{K})_1$, $\delta(\pi K)_{1/2}$ and $\delta(\pi K)_{3/2}$; the masses and widths of heavier class of scalar four-quark $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$, and hybrid mesons. For the nearest glue-rich scalar meson, S^*, which we will assign to $f_0(1710)$ [1] later, we parameterize the ratio of the asymptotic matrix elements of A_K to A_π as $Z = \langle K^+|A_{K^+}|S^*\rangle/\langle\pi^+|A_{\pi^+}|S^*\rangle$.

We now look for the values of the above parameters which reproduce the measured decay branching ratios for hadronic two body decays of charm mesons. To this, we take $f_+^{(\bar{K}D)}(0) = 0.74 \pm 0.03$ [34] as before. All the other parameters mentioned before are treated as adjustable ones with restrictions that a_1 and a_2 should not be very far from the ones estimated by the perturbative QCD [52], the non-resonant strong phases between -90° and 90°, four-quark meson masses not very far from the ones estimated in Ref. [29]. When we take

the following values of the parameters, we can reproduce the measured branching ratios for two body decays of charm mesons as seen in Table 4: the coefficients, $a_1 = 0.825$ and $a_2 = -0.159$ (*c.f.*, $a_1^{\rm BSW} \simeq 1.09$ and $a_2^{\rm BSW} \simeq -0.09$ at $\mu \simeq m_c$); the asymptotic matrix elements of $\tilde{H}_w$, $\langle\pi^+|\tilde{H}_w|D_s^+\rangle = 0.0641 \times 10^{-5}$ (GeV)^2; the parameters describing the pole contributions of the heavier class of four-quark $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$ mesons, the glue-rich scalar and the hypothetical hybrid mesons, $k_a^* = 0.0771$, $k_s^* = -0.0217$, $f_g = 0.0387$, $k_H = -0.014$, respectively, where we have assigned the $f_0(1710)$ with the mass $m_{f_0} = 1710$ MeV and the width $\Gamma_{f_0} = 125$ MeV [1] to the glue-rich scalar meson and have taken the ratio, $Z \simeq 1.56$, from the measured decay branching ratios of $f_0(1710)$; the relative phase between the factorized and non-factorizable amplitudes, $\delta = -19.9°$; the phases arising from non-resonant meson-meson interactions, $\delta_0(\pi\pi) = \delta_0(K\bar{K}) = 57.0°$, $\delta_1(K\bar{K}) = 58.7°$, $\delta_{1/2}(\pi K) = 84.4°$, $\delta_{3/2}(\pi K) = -26.7°$; the masses and widths of scalar non-$\{q\bar{q}\}$ mesons, $m_{\hat{\sigma}^*} = 1.514$ GeV, $m_{E^*_{\pi\pi}} = 2.164$ GeV, $m_{\sigma_H} = 2.012$ GeV (the iso-singlet scalar hybrid) with the mass differences, $\Delta_s = 0.1$ GeV and $\Delta_c = 1.3$ GeV, and $\Gamma_{[qq][\bar{q}\bar{q}]} = 0.198$ GeV, $\Gamma_{(qq)(\bar{q}\bar{q})} = 0.256$ GeV, $\Gamma_{\{q\bar{q}g\}} = 0.0456$ GeV. The above mass values of four-quark mesons are a little higher than the ones in Ref. [29] while the masses of hybrid mesons with a normal $J^{P(C)} = 0^{+(+)}$ have been much higher than the ones predicted by the covariant oscillator quark model [53]. To solve the puzzle, Eq.(14), the role of the glue-rich scalar $f_0(1710)$ and the four-quark $\hat{\sigma}^{s*}$ have been very important.

In summary we have studied charmed scalar mesons. Since the mass of the newly observed $D_{s0}^+(2.32)$ has been much lower than the ones calculated by using various models, many different assignments (in addition to the ordinary scalar $\{c\bar{s}\}$ meson or the chiral partner of D_s^+ prior to the observation) of it have been proposed. These models have extra scalar mesons in addition to the ordinary scalar $\{c\bar{q}\}$ mesons. Among these models, we have assigned the $D_{s0}^+(2.32)$ to the $I_3 = 0$ member, $\hat{F}_I^+$, of the iso-triplet $\hat{F}_I^+$'s which belong to the lighter class of four-quark $[cq][\bar{q}\bar{q}]$ mesons and have investigated the decay rates of the members of the same multiplet. As the consequence, we have predicted that the iso-triplet $\hat{F}_I$'s are narrow and the iso-doublet, $\hat{D}$, mesons are a little broader. However, another iso-doublet $\hat{D}^s$'s are around the threshold of the decay, $\hat{D}^s \to D\eta$, which has been expected to be the dominant decay of $\hat{D}^s$, so that its rate would be very small even if it is allowed. The iso-singlet $\hat{F}_0^+$ is extremely narrow since its main decay would proceed through iso-spin violating interactions. The exotic state $\hat{E}^0$ would decay through weak interactions if its mass is close to $m_{\hat{F}_I}$.

In addition to the $[cq][\bar{q}\bar{q}]$, we have the ordinary scalar mesons, D_0^* and D_{s0}^{*+}. Their masses have been expected to be around ~ 2.35 GeV and ~ 2.45 GeV, respectively. Comparing with the K_0^* which has been considered to be the ${}^3P_0$ $\{n\bar{s}\}$, we have obtained $\Gamma_{D_0^*} \sim (60-90)$ MeV and $\Gamma_{D_{s0}^*} \sim (40-70)$ MeV. Therefore, we expect that there coexist two scalar mesons, i.e., the four-quark $\hat{D}$ and the ordinary D_0^* in the region of the broad bump around 2.31 GeV in the $D\pi$ mass distribution which has been observed by the BELLE collaboration, and hope that the BELLE collaboration will reanalyze their data on the $D\pi$ mass distribution by using a model amplitude with at least two scalar meson poles. The strange counterpart, D_{s0}^{*+}, of the D_0^* is massive enough to decay into the DK final state so that it has been expected to be observed in the DK channel. It is also awaited that experiments with high luminosities and high resolutions will search for a scalar resonance with a mass $\sim (2.4-2.5)$ GeV and a width $\sim (60-90)$ MeV in the DK channel.

Four-quark mesons have been classified into various flavor multiplets with different $J^{P(C)}$ and color configurations. However, it is expected, from the results in Ref. [29], that the multiplets other than the low lying $[cq][\bar{q}\bar{q}]$ with $J^{P(C)} = 0^{+(+)}$ and dominantly $\bar{\mathbf{3}} \times \mathbf{3}$ of color $SU_c(3)$ considered above are much heavier than the low lying ones and the P-wave $\{c\bar{q}\}$'s, i.e., D_0^*, D_1^*, $D_1^{'*}$, D_2^* and D_{s0}^*, D_{s1}^*, $D_{s1}^{'*}$, D_{s2}^*. Therefore they do not disturb the known P-wave $\{c\bar{q}\}$ spectrum. Namely, only the low lying scalar $[cq][\bar{q}\bar{q}]$ mesons coexist with the ${}^3P_0$ $\{c\bar{q}\}$ mesons in the the $\sim (2.2-2.5)$ GeV region.

Finally, it has been discussed that four-quark mesons can play a very important role in hadronic weak decays of charm mesons, in particular, can solve the long standing puzzle, Eq.(14), in a overall consistent way. Therefore, hadronic weak interactions of charm mesons are intimately related to hadron spectroscopy and confirmation of the existence of four-quark mesons will open a new window of hadron physics, not only hadron spectroscopy but also hadronic weak interactions.

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